SLR-MM – 508

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2015
STATISTICS (Paper – III) (CGPA) (Old)
Linear Algebra

Day and Date : Friday, 20-11-2015 Total Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative :

i) Let V = {x, y, z, x, y, z∈ R} be a vector space then dimension of V is

A) 1 B) 2
C) 3 D) 0

ii) If A is an orthogonal matrix then

A) $A = A^T$ B) $A = A^{-1}$
C) $A = -A^T$ D) $A = A^2$

iii) Let A be an idempotent matrix. Then the value of $\underset{X}{\text{Max}} \frac{X^TAX}{X^TX}$ is ________

A) 1 B) 0
C) –1 D) ∞

iv) Let A and B be non-singular square matrices of the same order. Then which of the following is true ?

A) Rank (A) > Rank (B) B) Rank (A) < Rank (B)
C) Rank (A) ≠ Rank (B) D) Rank (A) = Rank (B)

v) Consider the following system of equations :

x + y = 3, x – y = 1, 2x + y = 5.

The above system has

A) Unique solution B) No solution
C) More than one solution D) None of these

P.T.O.

B) Fill in the blanks :

i) The rank of a $K \times K$ orthogonal matrix is ___________

ii) A superset of linearly dependent set of vectors is linearly ___________

iii) If the trace and determinant of a 2×2 matrix are 5 and 6, then smallest characteristic root is ___________

iv) Any square matrix can be written as sum of symmetric and ___________

v) If $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then M^{-1} = ___________

C) State whether following statements are **true** or **false** :

I) A matrix $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ is positive definite matrix.

II) Every matrix has a unique g-inverse.

III) The symmetric matrix A of the quadratic form $(x_1 + x_2)^2$ is $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

IV) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $a \neq 0$, then $G = \begin{bmatrix} \frac{1}{a} & 0 \\ 0 & 0 \end{bmatrix}$ is always a g-inverse of A.

(5+5+4)

2. a) Answer the following :

a) Define vector space with illustration.

b) Define inverse of a matrix. Prove that $(AB)^{-1} = B^{-1}A^{-1}$.

b) Write short notes on the following :

a) Elementry operations on a matrix.

b) Moore-Penrose (MP) inverse. **(6+8)**

3. a) Define (I) Dimension of a vector space (II) Basis of a vector space. Prove that any two bases of vector space contain same number of vectors.

b) Define linearly independent and dependent set of vectors. Examine whether the following set of vectors is linearly independent.
$a_1 = (1, 1, 2)$ $a_2 = (2, 2, 3)$ $a_3 = (1, 2, 3)$. **(7+7)**

4. a) Define and illustrate one example each, the folloiwng terms :

   I) Rank of a matrix

   II) Kroneckar product of two matrices

   III) Skew symmetric matrix.

   b) Let N be a non-singular matrix of order n partitioned as $N = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix}$, where N_{22} is a non-singular matrix of order m (m < n). Obtain inverse of N. **(6+8)**

5. a) Define g-inverse of matrix. Show that $\overline{A}$ is a g-inverse of A iff $A\overline{A}A = A$.

   b) Show that a system of linear equations $A\underline{X} = b$ is consistent iff $\rho(A|b) = \rho(A)$. **(6+8)**

6. a) Define characteristic roots and vectors of a matrix show that the characteristic vector corresponding to the distinct characteristic roots of real symmetric matrix are orthogonal.

   b) Prove Cayley-Hamilton theorem. Indicate how can be used to find inverse of a given matrix. **(7+7)**

7. a) Define a quadratic form. Give an example. Show that quadratic form is invariant under non-singular transformation.

   b) Reduce the following quadratic form $x_1^2 + 2x_2^2 + 3x_3^2 - 2x_1x_2 + 2x_2x_3 - 2x_1x_3$ to canonical form and determine whether it is definite or indefinite. **(7+7)**

Seat No.	

M.Sc. (Part – I) (Sem. – I) Examination, 2015
STATISTICS (Paper – III) (Old)
Linear Algebra

Day and Date : Monday, 20-4-2015 Max. Marks : 70

Time :11.00 a.m. to 2.00 p.m.

Instructions : *i) Attempt **any five** questions.*
*ii) Q.No. **(1)** and Q.No. **(2) compulsory**.*
*iii) Attempt **any three** questions from Q. No. **3** to Q. No. **7**.*
*iv) Figures to **right** indicates **full** marks.*

1. A) Select the correct alternative :

i) If $\underline{X}$ and $\underline{Y}$ are linearly independent, then $\underline{X} + \alpha\underline{Y}$ and $\underline{X} + \beta\underline{Y}$ are linearly dependent if

A) $\alpha = \beta$ B) $\alpha < \beta$ C) $\alpha > \beta$ D) $\alpha \neq \beta$

ii) The characteristic roots of a real symmetric orthogonal matrix are

A) 0 or 1 B) –1 or 1 C) 0 or –1 D) None of these

iii) The rank of $A = \begin{bmatrix} 4 & 0 & 0 \\ 6 & 6 & 12 \\ 4 & 4 & 8 \end{bmatrix}$ is

A) 2 B) 1 C) 3 D) None of these

iv) Let A be an idempotent matrix. Then the value of $\max\limits_{X} \dfrac{X'AX}{X'X}$ is

A) 0 B) 1

C) Cannot be determined D) None of these

v) The determinant and trace of 2×2 matrix A are 12 and 8 respectively, then characteristic roots are

A) 2 and 6 B) 3 and 4 C) 12 and 1 D) 8 and 1

B) Fill in the blanks :

i) If λ is characteristic root of A, then the characteristic root of (A + I) is ____

ii) The dimension of the vector space V = {(x, y, x + 2y) : x, y $\in$ R} is ______

iii) The rank of a K $\times$ K orthogonal matrix is ______

iv) The quadratic form $x_1^2 + x_2^2$ is ________ definite.

v) The system of equations 2x + 2y = 6, x – y = 1, 4x + 2y = 10 has ______ solution.

C) State **true** or **false** :

i) Moore Penrose (M – P) inverse is not unique.

ii) A matrix $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ is positive semidefinite matrix.

iii) P is an idempotent matrix if $P = P^2$.

iv) The g-inverse of (1, 1, 1) is $(1, 1, 1)^T$. **(5+5+4)**

2. a) i) Define inverse of matrix. Find the inverse of matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

ii) Obtain g-inverse of $\begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & 2 \\ 2 & 0 & 4 \end{bmatrix}$.

b) Write short notes on the following :

i) Row and column space of a matrix.

ii) Classification of a quadratic form. **(6+8)**

3. a) Define and illustrate giving one example each (i) Vector space (ii) Canonical form of a quadratic form.

b) Describe Gram-Schmidt orthogonalization process. Using this method obtain an orthogonal basis for R^2 starting with vector $a_1 = (2, 4)$ and $a_2 = (2, 8)$. **(7+7)**

4. a) Define rank of a matrix. Prove that rank (AB) $\leq$ min {rank (A), rank (B)}.

b) Let X and Y be n-component linearly independent vectors. Show that $X + \alpha Y$ and $X + \beta Y$ are also linearly independent if $\alpha \neq \beta$. **(7+7)**

5. a) Define (i) trace of a matrix (ii) symmetric matrix (iii) skew-symmetric matrix. Give an example each.

   b) Let A and B be two square matrices. Then prove or disprove AB and BA have the same characteristic roots. **(7+7)**

6. a) State and prove a necessary and sufficient condition for a system of linear equations AX = b to be consistent.

   b) Examine for the definiteness of the quadratic form (i) $4x_1^2 - 4x_1x_2 + x_2^2 + x_3^2$

   (ii) $\sum_{i=1}^{n} x_i^2$. **(7+7)**

7. a) Explain the spectral decomposition of a symmetric matrix. Give an illustration.

   b) Prove that a necessary and sufficient condition for a quadratic form $X'AX$ to be positive definite is that $\begin{vmatrix} a_{11} & \cdots & a_{1i} \\ \vdots & & \\ a_{i1} & \cdots & a_{ii} \end{vmatrix} > 0$ for i = 1, 2, . . .n. **(7+7)**

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SLR-JS-368

Seat No.	

Set P

M.Sc.(Semester - I) (CBCS) Examination Oct/Nov-2019
Statistics
LINEAR ALGEBRA

Day & Date: Tuesday, 05-11-2019 Max. Marks: 70
Time: 11:30 AM To 02:00 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Fill in the blanks by choosing correct alternatives given below. **14**

1) Eigen values of an idempotent matrix are -
   a) -1 or 1 b) 0 or 1
   c) 2 or 1 d) None of these

2) Let A be a square matrix, then A is said to be nilpotent if- for any positive integer k-
   a) $A^k = 0$ b) $A^k = I$
   c) $A^k = -1$ d) None of these

3) For a matrix N with 5 rows and 3 columns, $\rho(N)$ is rank of N then
   a) $\rho(N) \leq 5$ b) $\rho(N) \geq 3$
   c) $\rho(N) \leq 3$ d) $\rho(N) \geq 5$

4) Let B be any real matrix and A be its inverse then
   a) BA = I b) AB = I
   c) both a) and b) d) None of these

5) Eigen values of an upper triangular matrix are -
   a) Its diagonal elements b) off diagonal elements
   c) all zero d) None of the these

6) The vector $\begin{bmatrix}1\\0\\1\end{bmatrix}$ is an Eigen vector of the matrix $\begin{bmatrix}2 & 5 & 1\\1 & 7 & -1\\1 & 0 & 2\end{bmatrix}$ then corresponding Eigen value is -
   a) 0 b) 1
   c) 2 d) 3

7) Let V be a vector space of all functions $f(x)$ where $f: R \to R$
   Then which of the following are subspace of V-
   A. The constant function B. The function with $\lim_{x\to\infty} f(x) = 3$
   C. Function with $f(1) = 1$ D. Function with $f(0) = 0$
   a) A, B, C and D b) A and D only
   c) B,C and D only d) B and D only

8) The column space of a non-singular matrix N of order 3 has dimension -
   a) 3 b) less than 3
   c) greater than 3 d) None of these

9) A vector space is closed under the operation of _____.
   a) addition and scalar multiplication
   b) addition and subtraction
   c) Division and multiplication
   d) None of these

10) Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$ then A^{-1} = _______.
   a) $\frac{1}{2}\begin{bmatrix} 4 & -1 \\ -2 & 1 \end{bmatrix}$
   b) $\frac{1}{2}\begin{bmatrix} 4 & -2 \\ -1 & 1 \end{bmatrix}$
   c) $\frac{1}{2}\begin{bmatrix} 1 & 4 \\ -1 & -2 \end{bmatrix}$
   d) None of these

11) Which of the following is an elementary row operation?
   a) $R_i \leftrightarrow R_j$
   b) $k.R_i \rightarrow R_{i,} k \neq 0$
   c) $R_i + k.R_j \rightarrow R_{i,} i \neq j$
   d) All the above

12) M is negative definite matrix if and only if all of its Eigen values are -
   a) negative or positive
   b) non positive
   c) negative
   d) None of these

13) For a system of non-homogeneous equations $\mathrm{Ax} = \mathrm{b}$, it has solution if _____.
   a) $\rho(\mathrm{A}) = \rho(\mathrm{A} : \mathrm{b})$
   b) $\rho(\mathrm{A}) < \rho(\mathrm{A} : \mathrm{b})$
   c) $\rho(\mathrm{A}) \neq \rho(\mathrm{A} : \mathrm{b})$
   d) None of these

14) The quadratic form $2\mathrm{X}_1{}^2 + \mathrm{X}_2{}^2$ is -
   a) positive definite
   b) negative definite
   c) positive semi definite
   d) negative semi definite

Q.2 a) Answer the following (any four): 08
1) Define algebraic and geometric multiplicity.
2) What is matrix of the quadratic form $\mathrm{X}_1^2 - 2\mathrm{X}_2^2 - \mathrm{X}_1\mathrm{X}_1$?
3) Define Subspace. Give an illustration.
4) Define Kronekar product.
5) Define Eigen value and Eigen vector.

b) Write Notes on (Any Two) 06
1) Elementary matrix operations
2) Row space and column space of a matrix
3) Singular value decomposition

Q.3 a) Answer the following (Any two) 08
1) What is definiteness of a quadratic form?
2) Describe procedure of obtaining of system of Non-homogeneous linear equations?
3) How to obtain inverse of partitioned matrix?

b) Answer the following (Any One): 06
1) Prove that any given quadratic form can be transformed to a quadratic form which contains only square terms.
2) Show that rank of product of any two real matrices does not exceeds rank of either of the matrix.

Q.4 a) Answer the following (Any Two) 10
1) State and prove Cayley Hamilton theorem.
2) State and obtain necessary and sufficient condition for positive definiteness of a given quadratic form.
3) Define g-inverse of a matrix. Write procedure to obtain g-inverse.

b) Answer the following (Any One): **04**

1) Let X, Y and Z are linearly independent vectors. Examine whether U = X+Y, V = Y+Z and W = X+Z are linearly independent or not.
2) Write a short note on Spectral decomposition.

Q.5 Answer the following (Any Two) **14**

a) Prove that any two linearly independent vectors in R^2 can form basis for R^2.

b) Obtain A^3 and A^{-1} using Eigen value analysis, where $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

c) Obtain orthonormal basis from the vectors a = (2, 0, 3), b = (1, 1, 0) and c = (0, 2, 1) using Gram-Schmidt process of orthonomalization.

SLR-MM – 503

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2015
STATISTICS (Paper – III)
Linear Algebra (New CBCS)

Day and Date : Friday, 20-11-2015 Total Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q.No. **(1)** and Q. No. **(2)** are **compulsory.***
*3) Attempt **any three** from Q. No. **(3)** to Q. No. **(7)**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative :

1) Which of the following sets of vectors are linearly dependent ?

$S_1 = \{(1, 2), (3, 4)\}$, $S_2 = \{(1, 2), (3, 4), (5, 6)\}$,

$S_3 = \{(1, 2, 3), (3, 4, 5)\}$, $S_4 = \{(1, 2, 3), (0, 0, 0)\}$

a) S_1 only b) S_2 and S_4 c) S_1 and S_3 d) S_4 only

2) Let A be a matrix A^{-1} exists if and only if A is a __________ matrix.

a) Non-singular b) Square

c) Singular d) Real symmetric

3) The eigen values of a triangular matrix are __________

a) Zero and one

b) The diagonal elements of the matrix

c) The off-diagonal elements of the matrix

d) None of these

4) If G is a g-inverse of A, then __________

a) rank (G) $\leq$ rank (A) b) rank (G) $\geq$ rank (A)

c) rank (G) = rank (A) d) rank (G) $\leq$ rank (AG)

P.T.O.

5) The quadratic form $x_1^2 + x_2^2$ is __________

a) Positive definite
b) Negative definite
c) Positive semi-definite
d) Negative semi-definite **(1×5)**

B) Fill in the blanks:

1) The dimension of the vector space $V_3 = \{(x, x, y) : x, y \in (-\infty, \infty)\}$ is __________

2) A set of n + 2 vectors in n-dimensional Euclidean space is always linearly __________

3) Let A be an m×n matrix then the system of linear equations Ax = 0 has non-trivial solution if and only if __________

4) The eigen vectors of a symmetric matrix corresponding to different eigen values are __________

5) The matrix associated with the quadratic form $2x_1^2 + 3x_1x_2$ is ________ **(1×5)**

C) State **true** or **false** :

1) Let A and B be the two matrices. Then rank (A + B) $\leq$ min {rank (A), rank (B)}.

2) G inverse of a nonsingular matrix is unique.

3) All the eigen values of a non-singular matrix are non-zero.

4) Let p and q be the numbers of positive and negative d_i' s in the quadratic form $Q = \sum_{i=1}^{n} d_i x_i^2$, then Q is positive definite if and only if p = n. **(1×4)**

2. a) i) If X, Y, and Z are linearly independent vectors, examine whether U = X + Y, V = Y + Z, and W = X + Z are linearly independent.

ii) Prove or disprove that if λ is an eigen value of matrix A with corresponding eigen vector x then λ^m is an eigen value of A^m with corresponding eigen vector x for m = 2, 3, ... **(3+3)**

b) Write short notes on the following : **(4+4)**

i) Singular value decomposition

ii) Elementary row and column transformations of matrices.

3. a) Obtain orthonormal basis from the vectors a = (2, 0, 3), b = (1, 1, 0) and c = (0, 2,1) using Gram-Schmidt process of orthogonalization.

b) Show that any set of n linearly independent vectors in n-dimensional Euclidean space forms a basis for n-dimensional Euclidean space. **(7+7)**

4. a) Let A and B be m×n and n×p matrices, respectively. Show that rank (AB) $\leq$ min {rank(A), rank(B)}.

b) State and prove Caley-Hamilton theorem. **(7+7)**

5. a) Let $\lambda_1, \lambda_2, ..., \lambda_n$, be the characteristic roots of an n×n matrix A. Show that

$$|A| \prod_{i=1}^{n} \lambda_i \text{ and trace } (A) = \sum_{i=1}^{n} \lambda_i .$$

b) Show that if a real symmetric matrix A has eigen values 0 and 1 only then A is idempotent. **(7+7)**

6. a) Prove that matrix G is a g-inverse of matrix A if and only if AGA = A.

b) Consider a system of linear equations Ax = 0, where A is an m×n matrix of rank r(< n). Show that the number of linearly independent solutions to the system is n – r. **(7+7)**

7. a) Prove that the definiteness of a quadratic form is invariant under nonsingular linear transformation.

b) Reduce the following quadratic form to a form containing only square terms $x_1^2 + x_3^2 + 4x_1x_3 + 8x_2x_3.$ **(7+7)**

5. a) Describe any four tests for convergence of series.

   b) Show that the series $x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ converges absolutely for all values of x.

   c) Show that for any fixed value of x, $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$ is convergent. **(8+3+3)**

6. a) Define Riemann integral. Prove that every continuous function is integrable.

   b) Find the radius of convergence of the following series.

      i) $1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \ldots$

      ii) $x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ **(8+6)**

7. a) Find the minimum value of $x^2 + y^2 + z^2$ when $x + y + z = 3a$.

   b) Show that the function $f(x) = x^2$ is uniformly continuous on $[-1, 1]$.

   c) Test the convergence of $\int_0^1 \frac{dx}{\sqrt{1 - x^3}}$. **(6+4+4)**

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SLR-BP – 469

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2015
STATISTICS (Paper – III)
Linear Algebra (New)

Day and Date : Monday, 20-4-2015 Max. Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*

*2) Q.No. **1** and Q. No. **2** are **compulsory**.*

*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*

*4) Figures to the **right** indicate **full** marks.*

1. A) Select correct alternative :

i) The rank of $A = \begin{bmatrix} 2 & 0 & 0 \\ 3 & 3 & 6 \\ 2 & 2 & 4 \end{bmatrix}$ is

A) 1 B) 2 C) 3 D) None of these

ii) The characteristic of a real symmetric orthogonal matrix are

A) 0 and 1 B) – 1 and 1

C) – 1 and 0 D) None of these

iii) The quadratic form $X_1^2 - X_2^2$ is

A) Positive definite B) Negative definite

C) Indefinite D) None of these

iv) A square matrix A is called skew-symmetric matrix if

A) $A = A^T$ B) $A = A^{-1}$ C) $A = A^TA$ D) $A = -A^T$

v) Let $V = \{X, X, X \mid X \in R\}$ be a vector space then dimension of V is

A) 1 B) 2 C) 3 D) None of the above

P.T.O.

B) Fill in the blanks :

I) If $A_{n \times n}$ is a non-singular matrix, then rank (A) = ____________

II) The system of equation : $2x + 2y = 6$, $3x - y = 5$, $2x + y = 5$ has ____________ solution.

III) If the trace and determinant of a 2× 2 matix are 10 and 16, then the largest characteristic root is ____________

IV) The matrix A of the quadratic form $X_1^2 + X_2X_3$ is ____________

V) The trace of a matrix is ____________ of diagonal elements of a matrix.

C) State whether the following statements are **True** or **False** :

I) If A is a positive semidefinite matrix then $|A|$ is zero.

II) Let $A = [1, 2, 3]^T$ then $G = [1\ 0\ 0]$ is a g-inverse of A.

III) Moore-Penrose inverse is not unique.

IV) The symmetric matrix A of the quadratic form $(X_1 - X_2)^2$ is $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.

2. a) Answer the following : **(5+5+4)**

I) Discuss classification of quadratic form.

II) Define Moore-Penrose inverse and state its properties.

b) Write short notes on the following :

I) Choleskey decomposition.

II) Vector space and subspace. **(6+8)**

3. a) Explain linearly independent set of vectors. Let X and Y be n-component linearly independent vectors. Show that $X + \alpha Y$ and $X + \beta Y$ are also linearly independent if $\alpha \neq \beta \neq 0$.

b) Describe Gram-Schmidt orthogonalization process using this construct an orthonormal basis for the vector space spanned $\underline{a}_1$ and $\underline{a}_2$ as given below

$a_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $a_2 = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$. **(7+7)**

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2015
STATISTICS (Paper – VII)
Linear Models (New) (CGPA)

Day and Date : Thursday, 19-11-2015 Max. Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **(1)** and Q. No. **(2)** are **compulsory**.*
*3) Attempt **any three** from Q. No. **(3)** to Q. no. **(7)**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative :

1) In general linear model, $y = X\beta + \varepsilon$

a) $\text{rank}[X'X, X'y] = \text{rank}\ [X'X]$ b) $\text{rank}[X'X, X'y] \leq = \text{rank}\ [X'X]$

c) $\text{rank}[X'X, X'y] \geq = \text{rank}\ [X'X]$ d) $\text{rank}[X'X, X'y] < \text{rank}\ [X'X]$

2) In one-way ANOVA model $y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$; $i = 1, 2, ...k;\ j = 1, 2, ... n_i$, the dimension of estimation space is

a) $k - 1$ b) n_i c) $n_i - 1$ d) k

3) In two-way ANOVA model $y_{ij} = \mu + \alpha_i + \beta_i + \varepsilon_{ij}$; $i = 1, 2,...p;\ j = 1, 2, ... q$ the test statistic for testing the equality of β_j's has F distribution with .....d.f.

a) $(p - 1), (p - 1)(q - 1)$ b) $(q - 1), (p - 1)(q - 1)$
c) $(p - 1), pq - p - q + 2$ d) $(p - 1), pq - p - q - 2$

4) A balanced design is ___________ connected.

a) sometimes b) always c) never d) generally

5) For a BIBD with usual notation, $\lambda(v - 1) =$

a) $k(r - 1)$ b) $k(r + 1)$ c) $r(k + 1)$ d) $r(k - 1)$

(1×5)

P.T.O.

B) Fill in the blanks :

1) In general linear model $y = X\beta + \in$, the quantity $XS^{-}X'$ is ___________ under the choice of g-inverse of $S = X'X$.

2) In general linear model, $y = X\beta + \varepsilon, V(\lambda'\beta) =$ ____________

3) A connected block design can not be __________________

4) A block design is ______________ if and only if $CR^{-\delta}N = 0$.

5) The degrees of freedom of error SS in two-way without interaction ANOCOVA model with p rows, q columns, 1 observation per cell, and m covariate is ___________ **(1×5)**

C) State **true** or **false**.

1) The degree of freedom of error SS in two-way ANOVA with interaction model with p rows and q columns and with one observation per cell is one.

2) In general linear model, any linear function of the LHS of normal equations is the BLUE of its expected value.

3) BIBD is not orthogonal.

4) A connected design is always balanced. **(1×4)**

2\. a) i) Show that any solution of normal equations minimizes the residual sum of squares.

ii) Examine whether the following block design is connected.

$$B_1 = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \; B_2 = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \text{ and } B_3 = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix}.$$ **(3+3)**

b) Write short notes on the following :

i) Tuckey's test of non-additivity

ii) Dual of a BIBD. **(4+4)**

3. a) Prove that in general model $y = X\beta + \in$, the BLUE of every estimable linear parametric function is a linear function of the LHS of normal equations, and conversely, any linear function of the LHS of normal equations is the BLUE of its expected value.

   b) Prove that in general linear model $y = X\beta + \in$, a necessary and sufficient condition for the estimability of a linear parametric function $\lambda'\beta$ is that $\lambda' = \lambda' H$, where $H = S^{-}S, S = X'X$. **(7+7)**

4. a) Derive the test for testing the hypothesis of the equality of treatment effects in one-way ANOVA model.

   b) Describe two-way ANOVA without interaction model with one observation per cell and obtain the least square estimates of its parameters. **(7+7)**

5. a) Describe Tuckey's and Scheff's procedures of multiple comparisons.

   b) Describe ANOCOVA model is general and obtain the least square estimates of its parameters. **(7+7)**

6. a) Derive a test for testing a general linear hypothesis in a general linear model.

   b) prove that RBD is connected, orthogonal and balanced. **(7+7)**

7. a) State and prove a necessary and sufficient condition for orthogonality of a connected block design.

   b) Prove that in a BIBD, the number of blocks is greater than or equal to the number of treatments. **(7+7)**

SLR-ES-366

Seat No.	

Set P

M.Sc. (Semester – I) (CBCS) Examination March/April-2019
Statistics
LINEAR ALGEBRA

Day & Date: Saturday, 27-04-2019 Max. Marks: 70
Time: 12:00 PM To 02:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 Select the correct alternative. **14**

1) If columns of a square matrix A are orthogonal, then ______.
   a) A is singular
   b) rows of A are orthogonal
   c) A is idempotent
   d) A is symmetric

2) Which of the following sets of vectors are linearly dependent?
   P = {(2, 4), (1, 0)}, Q = {(1, 2, 3), (4, 5, 6)},
   R = {(2, 3), (3, 4), (5, 6)}, S = {(1, 1, 2), (0, 0, 0)}.
   a) Only P
   b) P and Q
   c) P,Q, and R
   d) All

3) Null space______.
   a) has dimension zero
   b) contains no vector
   c) is a vector space
   d) contains only one vector (0,0,….,0)

4) Row space and column space of a matrix______________.
   a) are the subspaces of a single common vector space
   b) always coincide
   c) are not vector spaces
   d) have the same dimension

5) (1, 2, 3)' has ______.
   a) has no g-inverse
   b) the unique g-inverse
   c) two g-inverses
   d) three g-inverses

6) If **G** is a g-inverse of matrix **A**, then _____.
   a) rank (**A**) ≤ rank(**G**)
   b) rank (**A**) ≥ rank(**A**)
   c) rank (**A**) = rank(**G**)
   d) rank (G**AG**) < rank(**A**)

7) Let A be a 3x3 matrix. A necessary and sufficient condition for existence of a non-trivial solution to the system of linear equations $Ax = 0$ is ______.
   a) rank (A) = 3
   b) rank (A) ≤ 3
   c) rank (A) ≤ 2
   d) rank (A) = 2

8) Elementary row operation ______.
   a) does not change rank of a matrix
   b) is essentially post-multiplying the given matrix by an elementary matrix
   c) is essentially pre-multiplying the given matrix by an orthogonal matrix
   d) none of A, B, C

9) The eigen values of a triangular matrix are______.
   a) the diagonal elements of the matrix
   b) the off-diagonal elements of the matrix
   c) zero and one
   d) none of A, B, C

10) Let **A** be a square matrix of order n. Then, the maximum number of linearly independent vectors in the eigen space of **A** corresponding to its eigen value λ is_____.
   a) $n - \mathrm{rank}(A)$ b) $n - \mathrm{rank}(A - \lambda I)$
   c) $n - \mathrm{rank}(\lambda A)$ d) $\mathrm{rank}(A - \lambda I)$

11) The characteristic polynomial of matrix A is λ^4 - 1. Then, A^4 = ______.
   a) cannot be known b) I
   c) A d) λA

12) A 2x2 matrix has 2 and 3 in the first row; 0 and 1 in the second row. Then, the characteristic polynomial p(λ) is______.
   a) $\lambda^2 - 3\lambda + 2$ b) $\lambda^2 + 2$
   c) $\lambda^2 + 3$ d) $\lambda^2 - 2\lambda + 3$

13) The quadratic form $x_1^2 + x_2^2$ is_____.
   a) positive definite b) negative definite
   c) positive semi-definite d) negative semi-definite

14) Caley-Hamilton theorem can be used to______.
   a) obtain inverse of a matrix
   b) determine definiteness of a quadratic form
   c) characteristic roots of a matrix
   d) none of A, B, C

Q.2 A) Answer the following. (Any four) **08**

1) What is inverse of $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$
2) Define G-inverse.
3) What is characteristic value problem?
4) Let $\boldsymbol{l}' = (l_1, l_2)$ and $x' = (x_1, x_2)$ then what is the matrix associated with the quadratic form $(\boldsymbol{l}'\boldsymbol{x})^2$?
5) State a necessary and sufficient condition for a real quadratic form to be positive definite.

B) Write Notes on. (Any Two) **06**

1) Vector space
2) Kronecker product
3) Algebraic and geometric multiplicities of a characteristic root of a matrix

Q.3 A) Answer the following. (Any two) **08**

1) Prove that any subset of a linearly independent set of vectors is linearly independent.
2) Show that rank of sum of two matrices cannot exceed sum of their ranks.
3) Prove or disprove that if λ is an eigen value of matrix **A** with corresponding eigen vector **x** then λ^m is an eigen value of $\mathbf{A}^m$ with corresponding eigen vector **x** for m = 2,3,…….

B) Answer the following. (Any one) **06**

1) Show that every basis for n-dimensional Euclidean space contains exactly n vectors.
2) Show that any quadratic form can be transformed to a form containing only square terms.

Q.4 A) Answer the following. (Any two) **10**

1) Find a g-inverse of $\begin{bmatrix} 2 & 2 & 4 \\ 2 & 5 & 8 \\ 1 & 7 & 1 \end{bmatrix}$
2) Show that if a real symmetric matrix **A** has eigen values 0 and 1 only, then **A** is idempotent.
3) Prove that the definiteness of a quadratic form is invariant under nonsingular linear transformation.

B) Answer the following. (Any one) **04**

1) Show that any square matrix can be expressed as the sum of a symmetric and a skew-symmetric matrices.
2) If **A** and **B** are square matrices of order n, show that rank (**AB**) ≥ rank (**A**) + rank (**B**) – n.

Q.5 Answer the following. (Any two) **14**

1) Given a basis $\{\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots\ldots, \boldsymbol{a}_n\}$ for n-dimensional space and a non-null vector $\boldsymbol{b}$ in n-dimensional space, show that if any vector $\boldsymbol{a_i}$ for which $\alpha_i \neq 0$ in the representation of $\boldsymbol{b}$ as $\boldsymbol{b} = \sum_{i=1}^{n} \alpha_i a_i$ is replaced from $\{\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots\ldots, \boldsymbol{a}_n\}$ by $\boldsymbol{b}$, then the new set is also a basis for n-dimensional space.
2) Let $\lambda_1, \lambda_2, \ldots.\lambda_n$, be the characteristic roots of an n×n matrix A. Show that $|\boldsymbol{A}| = \prod_{i=1}^{n} \lambda_i$ and trace $(\boldsymbol{A}) = \sum_{i=1}^{n} \lambda_i$
3) Obtain spectral decomposition of $\boldsymbol{A} = \begin{bmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{bmatrix}$ and hence find $\mathbf{A}^4$

SLR-MB – 605

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2016
STATISTICS (Paper – III)
Linear Algebra (New CBCS)

Day and Date : Saturday, 2-4-2016 Total Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

***Instructions** : 1) Attempt **five** questions.*
*2) Q.No. **(1)** and Q.No. **(2)** are **compulsory**.*
*3) Attempt **any three** from Q.No. **(3)** to Q.No. **(7)**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative :

1) Which of the following statements are true ?

I. A single null vector always forms a linearly dependent set of vectors.

II. A single non-null vector not necessarily form a linearly dependent set of vectors.

III. A set of vectors consisting of a null vector can be linearly independent.

A) Only I B) All I, II and III

C) I and III D) II and III

2) Let A and B be the two matrices. Then, __________

A) rank (A + B) $\leq$ rank (A) + rank (B)

B) rank (A + B) = rank (A) + rank (B)

C) rank (A + B) $\geq$ rank (A) + rank (B)

D) rank (A + B) $\leq$ min {rank (A), rank (B)}

3) The eigen values of 2×2 matrix A are 2 and 6. Then, __________

A) $|A| = 8$ B) trace (A) = 12

C) $|A| = 12$ D) trace(A) = 4

P.T.O.

4) __________ is not a g-inverse of [1 2 3].

A) $(1 \quad 0 \quad 0)'$ B) $(0 \quad 1/2 \quad 0)'$

C) $(0 \quad 0 \quad 1/3)'$ D) $(1/2 \quad 0 \quad 0)'$

5) The quadratic form $(x_1 + x_2)^2$ is __________

A) positive definite B) negative definite

C) positive semi-definite D) negative semi-definite **(1×5)**

B) Fill in the blanks :

1) Let A and B be the two matrices. Then, rank (AB) = rank(A) if B is a __________ matrix.

2) Let A be a square matrix of order n. The maximum number of linearly independent vectors in the eigen space of A corresponding to its eigen value λ is __________

3) The maximum number of linearly independent solutions to a system of linear equations Ax = 0, where A is a 3×5 matrix of rank 3 is ________

4) g-inverse of a ________ matrix is unique.

5) The matrix associated with the quadratic form x_1x_2 is ________ **(1×5)**

C) State **true** or **false**.

1) A subset of linearly dependent set of vectors can be linearly independent.

2) If λ is an eigen value of matrix A then $c\lambda$ is also an eigen value of A, where c is any constant.

3) Moore-Penrose inverse is also a g-inverse.

4) Let p and q be the numbers of positive and negative d_i's in the quadratic form $Q = \sum_{i=1}^{n} d_i x_i^2$, then Q is non-negative definite if and only if q = 0. **(1×4)**

2. a) i) Prove that a matrix is singular if and only if zero is one of its eigen value.

ii) Show that the system of linear equations Ax = 0 has non-trivial solution if and only if rank of A is less than the number of columns of A. **(3+3)**

b) Write short notes on the following.

i) Gram-Schmidt process of orthogonalization.

ii) Elementary row and column transformations of matrices. **(4+4)**

3. a) Prove that a set of vectors $\{a_1, a_2, ..., a_k\}$ is linearly dependent if and only if any vector in that set can be expressed as a linear combination of the rest.

b) Show that every basis for n-dimensional Euclidean space contains exactly n vectors. **(7+7)**

4. a) Find A^{-1} and A^5 using Caley-Hamilton theorem, where $A = \begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix}$.

b) Let A and B be m×n and n×p matrices, respectively. Show that rank(AB) $\geq$ rank(A) + rank(B) – n. **(7+7)**

5. a) Explain the computation of the inverse of higher order matrix by partitioning.

b) Obtain spectral decomposition of $A = \begin{bmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{bmatrix}$ and hence find A^4. **(7+7)**

6. a) Find g-inverse of matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 1 & 2 & 0 & 1 \end{bmatrix}$.

b) Show that if a real symmetric matrix A has eigen values 0 and 1 only then A is idempotent. **(7+7)**

7. a) Prove that the definiteness of a quadratic form is invariant under nonsingular linear transformation.

b) Prove that a real quadratic form $x'Ax$ in n variables is positive definite if and only if

$$g_i > 0, i = 1, 2, ..., n \text{ where } g_i = \begin{vmatrix} a_{11} & a_{11} & \cdots & a_{11} \\ a_{11} & a_{11} & \cdots & a_{11} \\ \vdots & \vdots & & \vdots \\ a_{11} & a_{11} & \cdots & a_{11} \end{vmatrix}.$$ **(7+7)**

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SLR-BP – 474

Seat No.	

M.Sc. (Part – I) (Sem. – I) Examination, 2015
STATISTICS (Paper – III) (Old)
Linear Algebra

Day and Date : Monday, 20-4-2015 Max. Marks : 70

Time :11.00 a.m. to 2.00 p.m.

Instructions : *i) Attempt **any five** questions.*
*ii) Q.No. **(1)** and Q.No. **(2) compulsory**.*
*iii) Attempt **any three** questions from Q. No. **3** to Q. No. **7**.*
*iv) Figures to **right** indicates **full** marks.*

1. A) Select the correct alternative :

i) If $\underline{X}$ and $\underline{Y}$ are linearly independent, then $\underline{X} + \alpha\underline{Y}$ and $\underline{X} + \beta\underline{Y}$ are linearly dependent if

A) $\alpha = \beta$ B) $\alpha < \beta$ C) $\alpha > \beta$ D) $\alpha \neq \beta$

ii) The characteristic roots of a real symmetric orthogonal matrix are

A) 0 or 1 B) –1 or 1 C) 0 or –1 D) None of these

iii) The rank of $A = \begin{bmatrix} 4 & 0 & 0 \\ 6 & 6 & 12 \\ 4 & 4 & 8 \end{bmatrix}$ is

A) 2 B) 1 C) 3 D) None of these

iv) Let A be an idempotent matrix. Then the value of $\max_{X} \frac{X'AX}{X'X}$ is

A) 0 B) 1

C) Cannot be determined D) None of these

v) The determinant and trace of 2 × 2 matrix A are 12 and 8 respectively, then characteristic roots are

A) 2 and 6 B) 3 and 4 C) 12 and 1 D) 8 and 1

P.T.O.

B) Fill in the blanks :

i) If λ is characteristic root of A, then the characteristic root of (A + I) is ____

ii) The dimension of the vector space $V = \{(x, y, x + 2y) : x, y \in R\}$ is ______

iii) The rank of a $K \times K$ orthogonal matrix is ______

iv) The quadratic form $x_1^2 + x_2^2$ is ________ definite.

v) The system of equations $2x + 2y = 6$, $x - y = 1$, $4x + 2y = 10$ has ______ solution.

C) State **true** or **false** :

i) Moore Penrose (M – P) inverse is not unique.

ii) A matrix $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ is positive semidefinite matrix.

iii) P is an idempotent matrix if $P = P^2$.

iv) The g-inverse of (1, 1, 1) is $(1, 1, 1)^T$. **(5+5+4)**

2. a) i) Define inverse of matrix. Find the inverse of matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

ii) Obtain g-inverse of $\begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & 2 \\ 2 & 0 & 4 \end{bmatrix}$.

b) Write short notes on the following :

i) Row and column space of a matrix.

ii) Classification of a quadratic form. **(6+8)**

3. a) Define and illustrate giving one example each (i) Vector space (ii) Canonical form of a quadratic form.

b) Describe Gram-Schmidt orthogonalization process. Using this method obtain an orthogonal basis for R^2 starting with vector $a_1 = (2, 4)$ and $a_2 = (2, 8)$. **(7+7)**

4. a) Define rank of a matrix. Prove that rank (AB) $\leq$ min {rank (A), rank (B)}.

b) Let X and Y be n-component linearly independent vectors. Show that $X + \alpha Y$ and $X + \beta Y$ are also linearly independent if $\alpha \neq \beta$. **(7+7)**

5. a) Define (i) trace of a matrix (ii) symmetric matrix (iii) skew-symmetric matrix. Give an example each.

   b) Let A and B be two square matrices. Then prove or disprove AB and BA have the same characteristic roots. **(7+7)**

6. a) State and prove a necessary and sufficient condition for a system of linear equations AX = b to be consistent.

   b) Examine for the definiteness of the quadratic form (i) $4x_1^2 - 4x_1x_2 + x_2^2 + x_3^2$

   (ii) $\sum_{i=1}^{n} x_i^2$. **(7+7)**

7. a) Explain the spectral decomposition of a symmetric matrix. Give an illustration.

   b) Prove that a necessary and sufficient condition for a quadratic form $X'AX$ to be positive definite is that $\begin{vmatrix} a_{11} & \cdots & a_{1i} \\ \vdots & & \\ a_{i1} & \cdots & a_{ii} \end{vmatrix} > 0$ for i = 1, 2, . . .n. **(7+7)**

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Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2014
STATISTICS (Paper – III)
Linear Algebra

Day and Date : Friday, 25-4-2014 Total Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative :

1) A set of vectors containing a null vector is ______________
 a) Not necessarily dependent b) Necessarily dependent
 c) Necessarily independent d) A vector space

2) Let A be a matrix. A^{-1} exists if and only if A is a ______________ matrix.
 a) Non-singular b) Square
 c) Singular d) Real symmetric

3) The eigen values of 2×2 matrix A are 2 and 6. Then
 a) $|A| = 8$ b) trace (A) = 12
 c) $|A| = 12$ d) trace (A) = 4

4) ______________ is not a g-inverse of [1 2 3].
 a) $(1\ 0\ 0)'$ b) $(0\ 1/2\ 0)'$
 c) $(0\ 0\ 1/3)'$ d) $(1/2\ 0\ 0)'$

5) The quadratic form $-x_1^2 - 2x_2^2$ is
 a) Positive definite b) Negative definite
 c) Positive semi-definite d) Negative semi-definite **(1×5)**

P.T.O.

B) Fill in the blanks :

1) A basis for n-dimensional Euclidean space contains _____________ vectors.

2) Let A be a square matrix of order n. The maximum number of linearly independent vectors in the eigen space of A corresponding to its eigen value λ is _____________

3) g-inverse of matrix A is unique if A is _____________

4) The matrix associated with the quadratic form x_1x_2 is _____________

5) A system of linear equations Ax = b is said to be homogeneous if _____________ **(1×5)**

C) State **true** or **false** :

1) A set of (n + 1) vectors in n-dimensional Euclidean space is linearly dependent.

2) If λ is an eigen value of matrix A then $c\lambda$ is also an eigen value of A, where c is any constant ?

3) Moore-penrose inverse is unique.

4) Let p and q be the numbers of positive and negative d_i's in the quadratic form $Q = \sum_{i=1}^{n} d_i x_i^2$, then Q is positive definite if and only if p = n. **(1×4)**

2. a) i) Examine whether the vectors a = (3, 5, –4), b = (2, 7, –8) and c = (5, 1, –4) are linearly independent.

ii) Prove that a matrix is singular if and only if zero is one of its eigen value. **(3+3)**

b) Write short notes on the following :

i) Elementary row and column transformations of matrices

ii) System of linear equations. **(4+4)**

3. a) Show that any set of n linearly independent vectors in n-dimensional Euclidean space forms a basis for n-dimensional Euclidean space.

   b) Show that any subset of size (n – 1) of the set of vectors n vectors $\{x_1, x_2 ..., x_n\}$ in n-dimensional Euclidean space is linearly independent, where

   $x_1 = (1, -1, 0, 0, ..., 0)$,
   $x_2 = (1, 0, -1, 0, ..., 0)$,
   $x_3 = (1, 0, 0, -1, ..., 0)$,
   $\vdots$
   $x_{n-1} = (1, 0, 0, ..., -1)$, and
   $x_n = (n - 1, -1, -1, ..., -1)$. **(7+7)**

4. a) Let A and B be $m \times n$ and $n \times p$ matrices, respectively. Show that rank (AB) $\leq$ min {rank (A), rank (B) }.

   b) Prove that row rank of a matrix is same as its column rank. **(7+7)**

5. a) Find A^{-1} and A^5 using Caley-Hamilton theorem, where $A = \begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix}$.

   b) Prove or disprove that if λ is an eigen value of matrix A with corresponding eigen vector x then λ^m is an eigen value of A^m with corresponding eigen vector x for m = 2, 3, ...

   c) Show that the eigen values of an idempotent matrix are either 0 or 1. **(6+4+4)**

6. a) Consider a system of linear equations Ax = 0, where A is an $m \times n$ matrix of rank r (< n). Show that the number of linearly independent solutions to the system is n – r.

   b) If matrix A is such that $A = A'A$, show that A is symmetric and idempotent.

   c) If G is g-inverse of matrix A, show that G_1 = GAG is also a g-inverse of A. **(7+4+3)**

7. a) Prove that the definiteness of a quadratic form is invariant under non-singular linear transformation.

   b) Examine whether the following quadratic form is positive definite

   $x_1^2 + x_2^2 + 2x_3^2 + 2x_2x_3$. **(7+7)**
