Seat No.	

B.Sc. (Part -III) (Semester - VI) Examination, December - 2016
STATISTICS
Probability Theory (Paper - XIII)
Sub. Code: 65864

Day and Date :Wednesday, 14- 12 - 2016 **Total Marks : 40**
Time : 12.00 noon to 2.00 p.m.

Instructions : 1) **All questions are compulsory.**
2) **Figures to the right indicate full marks.**

Q1) Select the correct alternative: **[8]**

a) If X_1, X_2, X_3 is a random sample (r. s.) from exponential distribution with $\theta = 3$ then prob. distribution of smallest order statistic is exponential with θ = ______________.

i) 5
ii) 9
iii) 8
iv) None of these

b) Let X_1, X_2, X_3 be a r. s. from U(0, 1) then the distribution of sample range is

i) $\beta_2(2,2)$
ii) $\beta_2(1,n)$
iii) $\beta_1(2,2)$
iv) $\beta_1(1,n)$

c) If $P(X_n = 0) = 1 - \frac{1}{n}, P(X_n = 1) = \frac{1}{n}, n = 1,2$ ______________ then

i) $X_n \xrightarrow{2} 1$
ii) $X_n \xrightarrow{2} 2$
iii) $X_n \xrightarrow{2} 0$
iv) None of these

P.T.O.

d) A sequence of random variables $\{X_n, n \geq 1\}$ is said to converge in distribution function to X if

i) $\lim_{n\to\infty} F_n(X) = 1$

ii) $\lim_{n\to\infty} F(X) = 0$

iii) $\lim_{n\to\infty} F_n(X) = 0$

iv) None of these

e) In a discrete Markov chain a state j is said to be accessible from state i if

i) $P_{ij}^{(n)} > 0$

ii) $f_{ij}^{(n)} > 0$

iii) $P_{jj}^{(n)} > 0$

iv) None of these

f) A state of Markov chain is said to be Ergodic if it is

i) null persistent and aperiodic

ii) non-null persistent and aperiodic

iii) null persistent and periodic

iv) non-null persistent and periodic

g) Traffic intensity in queuing model with arrival rate λ and service rate μ is

i) $\dfrac{\lambda}{\mu}$

ii) $\dfrac{\lambda}{\lambda+\mu}$

iii) $\dfrac{\mu}{\lambda}$

iv) None of these

h) The probability distribution of service time in queuing system is

i) Exponential

ii) Normal

iii) Poisson

iv) Geometric

Q2) Attempt any two of the following: **[16]**

a) Define order statistics for a r. s. of size n drawn from a continuous distribution. Let $X_1, X_2, \text{----} X_n$ be a r.s. drawn from U(0,1) then obtain the distribution of

i) minimum order statistic

ii) maximum order statistic

b) Let $\{X_n, n \geq 1\}$ be a Markov chain with states 0, 1, 2 and transition probability matrix (t.p.m)

$$\begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}$$

and initial prob. dist^n. is $P[X_0 = i] = \frac{1}{3}, \quad i = 0,1,2$ then find

i) $P[X_2 = 2, X_1 = 1 / X_0 = 2]$

ii) $P[X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2]$

iii) $P[X_1 = 1]$

c) State and prove weak law of large numbers for i.i.d. random variables with finite variance.

Q3) Attempt any four of the following: **[16]**

a) Obtain distribution function of i^{th} order statistic.

b) Let X_1, X_2, ---- X_n be a r.s. drawn from $f(x) = \bar{e}^{(x-\theta)}$, $x \geq \theta, \theta > 0$ show that $X_{(1)} \xrightarrow{P} \theta$.

c) Define the terms

   i) Recurrent state

   ii) Transient state

d) What is queue? Explain essential features of queuing system.

e) Explain queuing model M/M/1 using FCFS queue discipline.

f) Let $\bar{X}_n$ be the mean of a r.s. of size 100 drawn from $\chi^2_{50 d.f}$. Compute an approximate value of $P(49 < \bar{X}_n < 51)$ [Given $\Phi(1) = 0.84134$].

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